Q.1 **Evaluate** $\int\left(2x^{1/2}+3x^{1/3}-4x^{1/4}\right)dx$

Sol. $\int\left(2x^{1/2}+3x^{1/3}-4x^{1/4}\right)dx$

$$= 2\int x^{\frac{1}{2}}dx+3\int x^{\frac{1}{3}}dx-4\int x^{\frac{1}{4}}dx \quad \text{[Rewrite]}$$

$$= 2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+3\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}-4\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} \quad \text{[Integrate]}$$

$$= \frac{4}{3}x^{\frac{3}{2}}+\frac{9}{4}x^{\frac{4}{3}}-\frac{16}{5}x^{\frac{5}{4}}+c \quad \text{[Simplify]}$$

Q.2 **Evaluate** $\int\frac{2^x+3^x}{5x}dx$

Sol. $\int\frac{2^x+3^x}{5x}dx=\int\left(\frac{2^x}{5x}+\frac{3^x}{5^x}\right)dx$

$$=\int\left[\left(\frac{2}{5}\right)^x+\left(\frac{3}{5}\right)^x\right]dx$$

$$=\frac{\left(2/5\right)^x}{\ln\left(2/5\right)}+\frac{\left(3/5\right)^x}{\ln\left(3/5\right)}+c$$

Q.3 **Evaluate** $I=\int\frac{dx}{(3x-2)^2}$

Sol. Put $3x-2=t \Rightarrow 3dx=dt$, so that

$$I=\frac{1}{3}\int\frac{dt}{t^2}=\frac{1}{3}\int t^{-2}dt$$

$$=\frac{1}{3}\int\frac{t^{-2+1}}{-2+1}+c=-\frac{1}{3t}+c$$

$$=-\frac{1}{(3x-2)}+c$$

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Q.1 Evaluate the definite integral $\int_1^4 \frac{dx}{\sqrt{x}}$

Sol. We have $\int_1^4 \frac{dx}{\sqrt{x}} = \int_1^4 x^{-1/2} dx = \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^4$

$$= \left[2\sqrt{x}\right]_1^4 = 2\left(\sqrt{4} - \sqrt{1}\right)$$
$$= 2(2-1) = 2$$

Q.2 Evaluate the definite integral $\int_2^4 \frac{dx}{x^2 - 9}$

Sol. Recall

Thus $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left[\frac{x+a}{x-a}\right]$

$$\int_2^4 \frac{dx}{x^2 - 9} = \frac{1}{(2)(3)}\left[\log\left|\frac{x+3}{x-3}\right|\right]_2^4$$
$$= \frac{1}{6}[\log 7 - \log 5]$$
$$= \frac{1}{6} \log \frac{7}{5}$$

Q.3 Evaluate the definite integrals $\int_1^2 \frac{dx}{(x+1)(x+2)}$

Sol. $\int_1^2 \frac{dx}{(x+1)(x+2)} = \int_0^2 \left[\frac{1}{x+1} - \frac{1}{x+2}\right] dx$ [split into partial fractions]

$$= \left[\log\left|\frac{x+1}{x+2}\right|\right]_1^2 = \log\frac{3}{4} - \log\frac{2}{3}$$
$$= \log\left(\frac{9}{8}\right)$$

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(c) Evaluate the integral $\int \frac{x}{(x+1)(2x-1)}dx$.

Sol. We first resolve the integrand into partial fractions. Write

$$\frac{x}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$$

$\Rightarrow x = A(2x-1) + B(x+1)$

put $x = \frac{1}{2}$ and -1 to obtain

If $x = \frac{1}{2}$, then $\frac{1}{2} = B\left(\frac{1}{2}+1\right) \Rightarrow B = \frac{1}{3}$

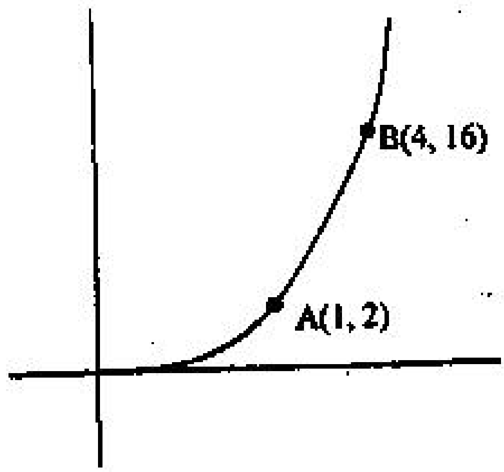
If $x = -1$, then $-1 = A(-3) \Rightarrow A = \frac{1}{3}$

Thus, $\int \frac{x}{(x+1)(2x-1)}dx = \frac{1}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{(2x-1)}$

$$= \frac{1}{3}\log|x+1| + \frac{1}{3}.\frac{1}{2}\log|2x-1| + c$$

$$= \frac{1}{3}\log|x+1| + \frac{1}{6}\log|2x-1| + c$$

(d) Find length of the curve $y = 2x^{3/2}$ from (1, 2) to (4, 16). [05]



Sol. We have

$$\frac{dy}{dx} = 3x^{1/2}$$

Required length

$$\int_1^4 \sqrt{1+9x}\,dx$$

$$= \left[\frac{(1+9x)^{3/2}}{9\left(\frac{3}{2}\right)}\right]_1^4 = \frac{2}{27}\left[37\sqrt{37} - 10\sqrt{10}\right] \text{ units.}$$

5. **(a) For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$.** [05]

Sol. If $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then the inequality holds trivially.

So let $|\vec{a}| \neq 0$, $|\vec{b}| \neq \vec{0}$. Then,

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})^2 = (\vec{a}+\vec{b}).(\vec{a}.\vec{b})$$

$$= \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}\vec{a} + \vec{b}\vec{b}$$